Verblunsky coefficients and polynomials in action in probability and statistics

Organizer: **Fabrice Gamboa,** Institute du Mathematiques de Toulouse and ANITI

This session aims to discuss and give an overview on some results for models arising from the stochastic world obtained by considering orthogonal polynomials.

Large deviations for random measures and sum rules

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A sum rule relative to a reference measure on the real line is a relationship between the reversed Kullback-Leibler divergence of a positive measure on the real line and some non-linear functional built on spectral elements related to this measure (see for example Killip and Simon 2003). In this talk, using only probabilistic tools of large deviations, we will explain how is possible to obtain sum rules. We will also review the last sum rules we have obtained. In particular, we recover the earlier result of Damanik, Killip and Simon (2010) when the reference measure is the (matrix-valued) semicircle law and obtain some new sum rules both for matrix-valued and real measures. This talk is based on joint works with F. Gamboa and J. Nagel (https://arxiv.org/abs/1811.06311,https://arxiv.org/abs/1604.06934,https://arxiv.org/abs/1601.08135).

On the circle, Kahane's Gaussian multiplicative chaos and circular random matrices match

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In this talk, I would like to advertise an equality between two objects from very different areas of mathematical physics. This bridges the Gaussian Multiplicative Chaos, which plays an important role in certain conformal field theories, and a reference model in random matrices. The main tool is an explicit description of canonical moments aka Verblunsky coefficients. On the one hand, in 1985, J.P Kahane introduced a random measure called the Gaussian Multiplicative Chaos (GMC). Morally, this is the measure whose Radon-Nikodym derivative w.r.t to Lebesgue is the exponential of a log correlated Gaussian field. In the cases of interest, this Gaussian field is a Schwartz distribution but not a function. As such, the construction of GMC needs to be done with care. In particular, in 2D, the GFF (Gaussian Free Field) is a random Schwartz distribution because of the logarithmic singularity of the Green kernel in 2D. Here we are interested in the 1D case on the circle. On the other hand, it is known since Verblunsky (1930s) that a probability measure on the circle is entirely determined by the coefficients appearing in the recurrence of orthogonal polynomials. Furthermore, Killip and Nenciu (2000s) have given a realization of the CBE, an important model in random matrices, thanks to random orthogonal polynomials of the circle. I will give the precise statement whose loose form is CBE = GMC. This is a joint work with J. Najnudel (https://arxiv.org/abs/1904.00578).

Localization of the continuous Anderson Hamiltonian in 1-d and its transition

Laure Dumaz*, Laboratoire du CEREMADE Dauphine*

We consider the continuous Schrödinger operator $- d^2/d^x^2 + B'(x)$ on the interval [0,L] where the potential B' is a white noise. We study the spectrum of this operator in the large L limit. We show the convergence of the smallest eigenvalues as well as the eigenvalues in the bulk towards a Poisson point process, and the localization of the associated eigenvectors in a precise sense. We also find that the transition towards delocalization holds for large eigenvalues of order L, where the limiting law of the point process corresponds to Sch_tau, a process introduced by Kritchevski, Valko and Virag for discrete Schrodinger operators. In this case, the eigenvectors behave like the exponential Brownian motion plus a drift, which proves a conjecture of Rifkind Virag. This is a joint work with Cyril Labbé.

Optimal Uncertainty Quantification of a risk measurement from a thermal-hydraulic code using Canonical Moments

Jérôme Stenger, EDF and Institut de Mathématiques de Toulouse

We study an industrial computer code related to nuclear safety. A major topic of interest is to assess the uncertainties tainting the results of a computer simulation. In this work we gain robustness on the quantification of a risk measurement by accounting for all sources of uncertainties tainting the inputs of a computer code. To that extent, we evaluate the maximum quantile over a class of distributions defined only by constraints on their moments. Two options are available when dealing with such complex optimisation problems: one can either optimise under constraints; or preferably, one should reformulate the objective function. We identify a well suited parameterisation to compute the optimal quantile based on the theory of canonical moments. It allows an effective, free of constraints, optimisation. This is a joint work with F. Gamboa, B. Iooss and M. Keller (https://arxiv.org/abs/1901.07903).