Sharpness of the phase transition in percolation

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In this minicourse, we will focus on the phase transition of Bernoulli percolation. Fix a parameter p. For each edge of the hypercubiclattice $\mathbb Z^d$, toss a biased coin: the edge is open with probability p and closed with probability $1-p$, independently of the other edges. We are interested in the connected components (called clusters) of the graph obtained by keeping only the open edges. As p increases, these clusters grow, and undergo a phase transition at a critical parameter p_c, above which an infinite cluster appears. In the 90's, this phase transition has been proved to be sharp:

- For $p<p_c$, all the clusters a very small. More precisely, the cluster of the origin has diameter larger than n with probability exponentially small in n.
- For $p>p_c$, there exists a unique infinite cluster, and the finite clusters are typically very small. Conditionally on being finite, the cluster of the origin has size larger than n with probability exponentially small in n.

In this minicourse, we will establish these results using modern techniques from the theory of Boolean functions. These methods are more robust than the previous approaches and apply to a large family of models in statistical mechanics.