

## *Random processes on networks and their limits*

Organizer: [Louigi Addario-Berry](#), MacGill University, CA

### **The scaling limit of a critical random directed graph**

[Christina Goldschmidt](#), University of Oxford, UK

We consider the random directed graph  $D(n, p)$  with vertex set  $\{1, 2, \dots, n\}$  in which each of the  $n(n-1)$  possible directed edges is present independently with probability  $p$ . We are interested in the strongly connected components of this directed graph. A phase transition for the emergence of a giant strongly connected component is known to occur at  $p = 1/n$ , with critical window  $p = 1/n + \lambda n^{-4/3}$  for  $\lambda \in \mathbb{R}$ . We show that, within this critical window, the strongly connected components of  $D(n, p)$ , ranked in decreasing order of size and rescaled by  $n^{-1/3}$ , converge in distribution to a sequence of finite strongly connected directed multigraphs with edge lengths which are either 3-regular or loops.

### **Recursive tree processes and mean-field limits of interacting particle systems**

[Anja Sturm](#), Georg-August-Universität Göttingen, Germany

In this talk we consider interacting particle systems, their description via graphical representations (stochastic flows) and their dual processes. We then focus on systems where the underlying lattice is given by the complete graph and consider the mean-field limit for which the number of vertices tends to infinity. We are not only interested in the mean-field limit of a single process, but also in how several coupled processes behave in the limit. These turn out to be closely related (dual in some sense) to recursive tree processes (RTP), which are generalizations of Markov chains with a tree-like time parameter, that were studied by Aldous and Bandyopadhyay in discrete time (alongside corresponding recursive distributional equations (RDE)). We illustrate our theory with a particle system with cooperative branching and deaths.

This is joint work with Tibor Mach and Jan Swart (Prague).

### **Interacting diffusions over random graphs and the role of sparsity**

[Roberto Imbuzeiro Oliveira](#), IMPA, Brazil

We report on joint work with Guilherme Reis (UFBA) and Lucas Stoleran (UCSD) on systems of diffusion processes whose interaction structure is given by a random graph. One example is the classical stochastic Kuramoto model over an Erdős-Rényi random graph. In the regime where the average degree of the graph diverges, we show that the model has the bulk behavior of the system obeys the same scaling limit and large deviations principle as in the complete graph. By contrast, when the degree converges to a constant, the limit of the model corresponds to a process on a Galton Watson tree (the "local weak limit" of the graphs). We will also discuss work in progress on equilibrium measures and mixing properties of systems in the two regimes.

### **Moderate deviation probabilities for subgraphs and other discrete structures**

[Simon Griffiths](#), PUC-Rio, Brazil

The study of deviation probabilities has a long history and in recent years many articles have focused on deviation probabilities for subgraph counts in the Erdős-Rényi random graph  $G(n, p)$ .

In this talk we consider the probability of moderately large deviations of subgraph counts in the alternative random graph model  $G(n, m)$  (also named after Erdős and Rényi), in which the number of edges is

fixed. We present the asymptotic rate associated with such deviations and deduce related bounds for the  $G(n,p)$  model.

Furthermore, we discuss related recent work in which we apply similar methods in a more general discrete framework which includes, for example, counts of arithmetic progressions.

[Based on joint work with Christina Goldschmidt and Alex Scott, and Gonzalo Fiz Pontiveros, Matheus Secco and Oriol Serra.]