Random discrete structures and statistical mechanics

Organizer: Christina Goldschmidt, Oxford University

This session will mostly be concerned with various processes which are built on (deterministic or random) graphs, in particular random walks, spin models and interacting particle systems.

Hipster random walks

Louigi Addario-Berry, McGill University

We study the behaviour of lazy random walks whose level of motivation to make a step is determined by the current popularity of the location where they find themselves.

Random graphs on the circle

Omer Angel, University of British Columbia

It is long known that two copies of the infinite Erdos-Renyi graph G(infinity,p) are almost surely isomorphic. The resulting graph is called the Rado graph. If the vertices are in a metric space and only nearby vertices may be connected, a similar result may or may not hold, depending on fine details of the underlying metric space. I will present new results, joint with Yinon Spinka, in the case where the metric space is a circle.

Algorithms at Low Temperatures

Will Perkins, University of Illinois at Chicago

There are two natural computational tasks associated to a statistical mechanics spin model on a graph: sampling from the model and approximating its partition function (the counting problem). The vast majority of known efficient sampling and counting algorithms are effective in the high temperature (weak interaction) regime of the given model, and for many structured families of graphs there are negative results known at low temperatures, the slow mixing of Markov chains. I will describe a systematic method for developing efficient counting and sampling algorithms at low temperatures in models with a bounded number of ground states. These algorithms use techniques from statistical physics (abstract polymer models, the cluster expansion, Pirogov-Sinai theory) that have classically been used to understand the probabilistic details of phase transitions and to prove slow mixing results for Markov chains. Based on joint work with Tyler Helmuth, Guus Regts, Matthew Jenssen and Peter Keevash.

Local Dynamics of Interacting Particle Systems on Large Sparse Graphs

Kavita Ramanan, Brown University

Models of large systems of homogeneous interacting particle systems on a (possibly random) graph in which the infinitesimal evolution of each particle depends on its own state and the empirical distribution of the states of neighboring particles arise in a variety of applications, ranging from engineering to physics. When the graph is a clique, it is well known that the dynamics of a typical particle converges in the limit, as the number of vertices goes to infinity, to a nonlinear Markov process, often referred to as the mean-field limit. In this talk, we focus on the complementary case of dynamics on sparse graphs, introduce motivating examples, identify limits of the empirical measure process and characterize the limiting marginal dynamics of a typical particle for certain classes of graphs. Our results also apply to more general history-dependent pure jump processes that arise in applications. This is based on joint work with Ankan Ganguly.