Average Gromov hyperbolicity and the Parisi ansatz

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Gromov hyperbolicity of a metric space measures the distance of the space from a perfect tree-like structure. The measure has a "worst-case" aspect to it, in the sense that it detects a region in the space which sees the maximum deviation from tree-like structure. I will speak about an "average-case" version of Gromov hyperbolicity, which detects whether the most of the space, with respect to a given probability measure, looks like a tree. The main result is that if this average hyperbolicity is small, then the space can be approximately embedded in a tree. The proof uses a weighted version of Szemeredi's regularity lemma from graph theory. As an application, I will give a construction of hierarchically organized pure states in any model of a spin glass that satisfies the Parisi ultrametricity ansatz. (Joint work with Leila Sloman.)