Random matrices

Organizer: Octavio Arizmendi, CIMAT, Mexico

Abstract
Random Matrix Theory is nowadays a prominent and very active area in probability, with important applications to other areas in science and engineering such as information theory, machine learning, wireless communications, statistical mechanics, number theory and functional analysis. This session plans to bring speakers from different countries in Latin America, United States and Europe, working in diverse aspects of Random Matrix Theory. This include limiting behavior in the edge of the spectrum, local limit laws, non-hermitian models, Free probability and its applications such as neural networks.

Conferences:

Asymptotic behavior of diffusion processes associated to beta-ensembles
Jose Alexander Ramirez Gonzalez, Universidad de Costa Rica, Costa Rica

Convergence to Brown´s spectral measure for polynomials in Ginibre matrices
Nicholas Cook, Stanford University, USA

The celebrated circular law provides the limiting empirical spectral distribution (ESD) for a sequence $(N^{-1/2}X_N)_{N\geq 1}$ of $N \times N$ matrices, where each X_N has iid centered entries of variance one. For the case of complex Gaussian entries (Ginibre matrices) this can be verified using the explicit joint density of eigenvalues, as was done by Ginibre in the 1960s. For the general case, a Hermitization strategy was introduced by Girko in the 1980s, but a fully rigorous implementation was not achieved until work of Bai in the 90s and, in full generality, by Tao and Vu in the 2007. The most technical aspect of these works was to obtain quantitative control on the pseudospectrum of X_N.

In this talk I will present recent progress on the more general problem of establishing the limiting ESDs for polynomials in a fixed number m of independent Ginibre matrices. A prediction for the limiting distribution is provided by the Brown measure for the polynomial evaluated at m freely independent circular operators in a von Neumann algebra; this measure is obtained by a limiting version of the Hermitization procedure. Even in the Gaussian setting no explicit formulas are available, and the most challenging task is to control the pseudospectrum

Based on joint work with Alice Guionnet and Jonathan Husson

Traffic independence and Freeness over the diagonal
Camille Male, CNRS, Université de Bordeaux, France

Traffic probability is a generalization of free probability introduced by myself in 2011 where the algebra is assumed to have more structure. They are designed for the analysis of random matrices invariant by conjugation by permutation matrices, which are not generically asymptotically free. The notion of traffics comes with its own notion of independence, which encodes the usual notions of non-commutative independence.

The purpose of this talk is to review two applications of this notion in the context of operator-valued free probability over the diagonal matrices: the asymptotic freeness over the diagonal of independent permutation invariant matrices, and the analysis of second-order distributions of certain Wigner matrices.

Analysis of artificial neural networks: old and new random matrix theory perspectives
Mario Díaz Torres, CIMAT, Mexico

In the last decades, the so-called neural networks (NNs) have achieved many successes across a wide variety of learning problems. As a consequence, there is an increasing interest in understanding how and why they work. Given the complexity of this task, oftentimes theoreticians rely on the analysis of simple NNs to gain intuition about the behavior of more complex ones. Ideally, this intuition then leads to heuristics that could be used by practitioners. In this talk, we discuss some success stories for which random matrix theory proved to be a key component of the analysis. We finish discussing work in progress devoted to extend some of these approaches to handle a wider range of NNs.