Bayesian Uncertainty Quantification in systems of differential equations models

Organizer: J Andrés Christen, CIMAT-CONACYT, Mexico

In the study of models arising from systems of ordinary or partial differential equations usually there are unknown parameters that need to be inferred from data. This is commonly known as Inverse Problems and in the past few years it has been accepted as a statistical problem. Moreover, providing a probabilistic, Bayesian, solution to Inverse Problems has been recognized as most suited to quantify the uncertainty in our inferences [1, 2]. This broadly refers to Bayesian UQ and has seen a strong growth in interest from large numbers of groups in academia and in industry, with applications in many fields of science and technology.

There are, none the less, a series of issues in this Bayesian inference problem that make it specially difficult and particularly challenging, both mathematically and computationally speaking.

A particular difficulty separating this inference problem from others is that the regressor model, eg. the PDE model, commonly cannot be computed directly but a numerical approximation is needed. Such numerical method involves an error and also involves heavy computing times, as it is the case with FEM methods for solving PDEs. This has two clear consequences: first, these numerical errors may be propagated into the posterior distribution and, second, the evaluation of the posterior can be very CPU costly. An other important aspect of Bayesian UQ is that the definition of the prior distribution is commonly non-standard and complex since in many cases the quantities of interest are functions that themselves need to be discretized and given a prior. Moreover, the resulting theoretical posterior distribution is only represented numerically in our computers through a discretization and we require consistency results assuring that the posterior is well defined and the corresponding discretization leads to a numeric posterior with sufficient error control. Last, but not least, we also require very efficient Monte Carlo methods to simulate from the posterior, taking into consideration the discretization error, the multidimensionality and the highly CPU costly evaluation of the posterior.

In this session we will present an introduction to this multidisciplinary subject with a first overview talk and also overview talks on research topics in Bayesian UQ in numerical analysis of PDEs, discretization error control, modeling, prior distributions and Monte Carlo methods, dimensionality reduction, several examples and one case study from Bayesian inversion methods for seismic data in the pacific shore of Mexico.

[1] Kaipio, Jari, Somersalo, E. (2005), ``Statistical and Computational Inverse Problems'', Springer.
[2] Fox, C and Haario, H and Christen, J A (2013) ``Inverse problems'', In: Bayesian Theory and Applications, Chapter 13, Ed: Damien, P et al, Oxford University Press.

Overview of Bayesian Uncertainty Quantification
Eduardo Gutiérrez-Peña, IIMAS, Universidad Nacional Autónoma de México, CU-UNAM

Inverse problems involving noisy observations are in fact problems of statistical inference. The Bayesian approach to solving inverse problems not only provides regularized estimates of the unknown parameters in a model, but also allows the researcher to assess and describe the overall uncertainty about these parameters. In addition, the Bayesian approach provides the tools to adequately propagate this uncertainty in order to make inferences about the quantities of actual interest. In this talk we shall give an overview of the Bayesian approach to regularization and uncertainty quantification in the context of generalized linear models.

FEM in Bayesian UQ
Antonio Capella, IMATE, Universidad Nacional Autónoma de México, CU-UNAM
Abstract: TBA

Posterior error control in Bayesian UQ
Marcos Capistrán, Centro de Investigación en Matemáticas-CONACYT (CIMAT-CONACYT).

We generalize the results of [1] on expected Bayes factors (BF) to control the numerical error in the posterior distribution to an infinite dimensional setting when considering Banach functional spaces and now in a prior setting. The main result is a bound on the absolute global error to be tolerated by the Forward Map numerical solver, to keep the BF of the numerical vs. the theoretical model near to 1, now in this more general setting, possibly including a truncated, finite dimensional approximate prior measure. In so doing we found a far more general setting to define and prove existence of the infinite dimensional posterior distribution than that depicted in, for example, [2]. Discretization consistency and rates of convergence are also investigated in this general setting for the Bayesian inverse problem.

[1] Capistrán, Marcos A., J. Andrés Christen, and Sophie Donnet. "Bayesian analysis of ODEs: solver optimal accuracy and Bayes factors." SIAM/ASA Journal on Uncertainty Quantification 4, no. 1 (2016): 829-849.
[2] Stuart, Andrew M. "Inverse problems: a Bayesian perspective." Acta numerica 19 (2010): 451-559.

Bayesian UQ in geophysics applications in Mexico
J. Cricelio Montesinos-López, Centro de Investigación en Matemáticas-CONACYT (CIMAT-CONACYT)

We present a case study for Bayesian inversion using seismic data of the Pacific shore of Mexico. The data corresponds to the Guerrero 2006 Slow Slip Event (SSE), with the observed displacements in 15 continuous GPS stations.
The objective is to determine slip along the subduction interface and analyze in detail the spatial evolution

of the slip with depth. Under a Bayesian framework, we assume Gaussian distributions for the error model and a Multivariate Truncated Normal (MTN) distribution for the prior distribution of the slip vector is considered.

The resulting posterior distribution is also MTN. To sample from the posterior, we propose an efficient algorithm based on optimal direction Gibbs to generate samples of the MTN distribution.