

Random dynamical systems with jumps II

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Abstract

Random events occur in nature throughout our everyday experiences. Taking stochastic effects into account is of central importance for the development of mathematical models of complex phenomena under uncertainty arising in applications. Macroscopic models in the form of differential equations for these systems contain randomness in many ways, such as stochastic forcing, uncertain parameters, random sources or inputs, and random initial and boundary conditions. The theory of random dynamical systems and stochastic differential equations provides fundamental ideas and tools for the modeling, analysis, and prediction of complex phenomena. The aim of this contributed session is to showcase new developments on the theory of random dynamical systems whose driven noise has a jump structure. The three speakers on this session will deal with different aspects of stochastic differential equations driven by Lévy processes. This is the second part of an already submitted session.

Stochastic n-point bifurcations for Markovian dynamics

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Brownian flows of diffeomorphisms are known to be rigid in the sense that any ω -wise invariant measure of the flow is uniquely determined by the usual invariant measure of the respective 1 and 2-point motion. For general Markovian systems this turns out to be false. In order to quantify this defect, we introduce the notion of a stochastic n-point bifurcation which provides new information about the random dynamics. We construct several classes of examples already over finite spaces including the minimal example where this phenomenon occurs.

This is a joint work with M. Hoegele.

Large Deviations for Lévy processes

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We illustrate through two examples the robust weak convergence approach to large deviations theory. The first problem we address is the first exit time problem for a perturbed dissipative dynamical system in small intensity by a pure jump process. In order to design a large deviations principle, the size of the jumps must be tuned by a small parameter in inverse proportion to the rate they occur. Within that framework we design the Freidlin-Wentzell routemap in order to solve the Kramers problem: we express the law and the mean of the exit time and we characterize the locus of exit in terms of a control problem associated to the large deviations that those objects obey. The second problem concerns the moderate deviations regime of a multiscale stochastic system with jumps where the slow component exhibits extrinsic memory from the underlying deterministic dynamics. If time permits, we will discuss ongoing work on further strategies to study the dynamics of perturbed systems with stochastic intrinsic memory and how Markovianity can therefore be bypassed in the analysis.

Synchronization of stochastic mean field networks of Hodgkin-Huxley neurons with noisy channels

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We are interested in a mathematical model of the collective behavior of a fully connected network of finitely many neurons, when their number and when time go to infinity. We assume that every neuron follows a

stochastic version of the Hodgkin-Huxley model, and that pairs of neurons interact through both electrical and chemical synapses, the global connectivity being of mean field type. When the leak conductance is strictly positive, we prove that if the initial voltages are uniformly bounded and the electrical interaction between neurons is strong enough, then, uniformly in the number of neurons, the whole system synchronizes exponentially fast as time goes to infinity, up to some error controlled by (and vanishing with) the channels noise level. Moreover, we prove that if the random initial condition is exchangeable, on every bounded time interval the propagation of chaos property for this system holds (regardless of the interaction intensities). Combining these results, we deduce that the nonlinear McKean-Vlasov equation describing an infinite network of such neurons concentrates, as time goes to infinity, around the dynamics of a single Hodgkin Huxley neuron with chemical +neurotransmitter channels. Our results are illustrated and complemented with numerical simulations. Based on joint work with Mireille Bossy (INRIA Sophia-Antipolis) and Hector Olivero (University of Valparaso) appeared in Journal of Mathematical Biology.