

## Branching structures

Organizer: Sandra Palau, IIMAS-UNAM, Mexico

### Abstract

The purpose of this session is to highlight recent developments on the topic of branching processes and branching structures. They constitute a fundamental topic in the theory of stochastic processes and are very interesting from the theoretical point of view as well as for its various applications. This area of probability had experienced an increasing interest in recent years in Latin America, and worldwide, since many complex systems such as Gaussian Free Field, Random Planar maps, biological structures, etc; use a lot of techniques which are currently developed in this area. In this session, we propose a diversity of topics related to Galton-Watson in continuous time, Branching Brownian motion, Continuous state branching processes and superprocesses. Two of the speakers are from the region and the other two speakers have been interacting with many researchers in Latin-America.

### ***A strong law of large numbers for supercritical BBM with absorption***

Santiago Saglietti, Israel Institute of Technology, Israel

We consider a (one-dimensional) branching Brownian motion process with a general offspring distribution having at least two finite moments, in which all particles have a drift towards the origin and are immediately killed if they reach it. It is well-known that if and only if the branching rate is sufficiently large, the population survives forever with a positive probability. We show that throughout this super-critical regime, the number of particles inside any given set normalized by the mean population size converges to an explicit limit, almost surely and in  $L^1$ . As a consequence, we get that, almost surely on the event of survival of the branching process, the empirical distribution of particles converges weakly to the (minimal) quasi-stationary distribution associated with the Markovian motion driving the particles. This proves a result of Kesten from 1978, for which no proof was available until now. Joint work with Oren Louidor.

### ***Quasi-Stationary Distribution for the discrete-state continuous-time branching process with logistic growth***

Maria Clara Fittipaldi, UNAM, Mexico

The discrete-state continuous-time logistic branching process defined by Lambert describes the evolution of a population whose dynamics are driven by the following rules:

- each particle gives birth at rate  $\rho$ , and the number of offspring born is  $k$  with probability  $\pi_k / \rho$ ;
- each particle dies naturally at rate  $d$ ;
- each particle selects another fixed particle at rate  $c$  and kills it.

The binary logistic branching process can be studied using the theory of birth-death process, and in this case there is a unique quasi-stationary distribution. In the general case, although it is known that the process reaches extinction almost surely, the existence of quasi-stationary distribution is still an open question. To address this problem, we study some properties of the extinction time and we develop an ad hoc version of the renewal dynamical approach given by Ferrari, Kesten, Martinez and Picco. This is a joint work with A. Christen (PUCV, Chile).

### ***Genealogies of samples from Galton-Watson processes***

Simon C. Harris, University of Auckland

Consider a continuous-time Galton-Watson (GW) branching process. Conditional on the population surviving until some large time  $T$ , take a sample of particles from those alive. What does the ancestral tree drawn out by this sample look like? Some special cases were known, eg. Durrett (1978), O'Connell (1995), Athreya (2012), but we will discuss an approach behind some more recent advances for near-critical GW processes. Our key tool is a change of measure involving  $k$  distinguished particles (known as spines) that decouples certain dependencies to permit some explicit computations. This is based on joint works with M.Roberts, S.Johnston, and J.C.Pardo-Millan.

### ***Superdiffusions with super-exponential growth: construction, mass and spread***

Janos Englander, University of Colorado, USA

Superdiffusions with mass creation (potential) terms that are 'large functions' are studied. When the potential term is large, the generalized principal eigenvalue is typically infinite and the usual machinery, including martingale methods and PDE as well as other similar techniques cease to work effectively, both for the construction and for the investigation of the large time behavior of superdiffusions. We show how to resolve these issues, using, among other things, a generalization of the Fleischmann-Swart 'Poisson-coupling,' linking superprocesses with branching diffusions, and the introduction of a new analytic concept: the ' $p$ -generalized principal eigenvalue.' Precise growth rates are achieved too in concrete cases. This is joint work with Z.-Q. Chen (Seattle).