Propagation of Chaos

Organizer: María Clara Fittipaldi
Departamento de Matemáticas, Facultad de Ciencias, UNAM, México.

This contributed session will be focused on the current advances in the study of propagation of chaos and mean field interacting particle models applied to different areas, such as statistical mechanics, population genetics and queueing theory. This session will also provide a forum for discussion about the connections between the various methods used in these fields.

Thermostated Kac model with rescaling

Roberto A. Cortez Milán, Departamento de Matemáticas, Universidad Andrés Bello, Chile.

We consider the thermostated Kac model, which describes the evolution of the velocity distribution of particles in a one-dimensional spatially homogeneous caricature of a gas, represented as a large collection of identical particles undergoing random energy-preserving binary collisions and also interactions against an external thermostat.
In this work we introduce a rescaling mechanism on the model, which has the effect of restoring the total energy, and produces an additional drift term in the associated kinetic equation. We prove convergence towards a unique equilibrium distribution, which exhibits properties that can differ from the classical Gaussian equilibrium.
We also study a finite N -particle system approximation, and prove that it satisfies the propagation of chaos property: as $N \to \infty$, the empirical measure of the system converges to the solution of the kinetic equation. Joint work with Hagop Tossounian.

Quantitative propagation of chaos for Kac particle systems and application to Boltzmann equation

Joaquin Fontbona Torres, Departamento de Ingeniería Matemática - Centro de Modelación Matemática, Universidad de Chile, Chile.

A Family of Brunet-Derrida type branching-selection particle systems: hydrodynamic limit and asymptotic velocity.

Pablo Groisman, Departamento de Matemática, Facultad de Ciencias Exactas y Naturales, Universidad de Buenos Aires, Argentina.

We will introduce a family of systems of particles evolving in the real line in which particles branch and die under pretty general rules. We will conjecture a hydrodynamic equation for this model as the number of particles goes to infinity and prove the convergence to this equation for some particular cases. These cases are sufficient to obtain the asymptotic velocity of the system for general rules. For a large class of rules, the asymptotic velocities depend only on the branching rate of particles close to the rightmost one.

Convergence properties of many parallel servers under power-of-D load balancing

Matthieu Jonckheere, Instituto del Cálculo, Universidad de Buenos Aires - CONICET, Argentina.

We consider a system of N queues with decentralized load balancing such as power-of-D strategies(where D may depend on N) and generic scheduling disciplines. To measure the dependence of the queues, we use the clan of ancestors, a technique coming from interacting particle systems. Relying in that analysis we prove quantitative estimates on the queues correlations implying propagation of chaos for systems with Markovian arrivals and general service time distribution. This solves the conjecture posed by Bramsom et. al. concerning the asymptotic independence of the servers in the case of processor sharing policy. We then proceed to prove asymptotic insensitivity in the stationary regime.
Joint work with Maria Clara Fittipaldi, Sergio I. Lopez Ortega (UNAM).